The region bounded by $y = x^2 - 9$, $y = 2(x-5)$ and $y = 0$ is revolved around the line $y = 10$. **SCORE: _____ / 8 PTS**

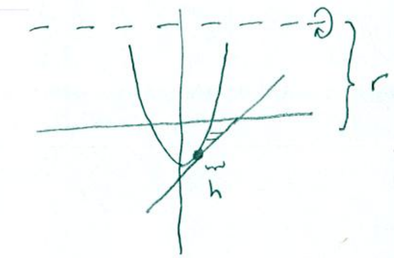
Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.

$x^2 - 9 = 2x - 10$

$\underbrace{x^2 - 2x + 1 = 0}_{}$ 1

$x = 1 \longrightarrow y = -8$

1 $\underbrace{2\pi}_{} \int_{\underbrace{-8}_{1}}^{\overbrace{0}^{1}} \underbrace{(10-y)}_{1} \underbrace{\left(\frac{y}{2} + 5 - \sqrt{y+9}\right)}_{3} dy$



Find the area bounded by the curves $y = 4x^3$ and $y = 16x^2 - 12x$. **SCORE: _____ / 8 PTS**

NOTE: Your final answer must be a number, not an integral nor sum of integrals.

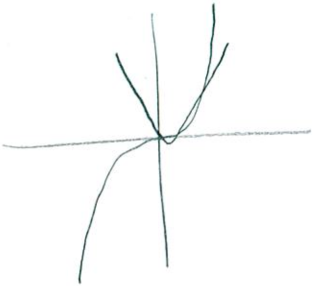
$4x^3 = 16x^2 - 12x$

$4x^3 - 16x^2 + 12x = 0$

$4x(x^2 - 4x + 3) = 0$

$4x(x-1)(x-3) = 0$ 1

$x = 0, 1, 3$



1 1 1 1

$\left[\int_0^1 (4x^3 - (16x^2 - 12x))\,dx + \int_1^3 (16x^2 - 12x - 4x^3)\,dx\right]$

$= \left(x^4 - \frac{16}{3}x^3 + 6x^2\right)\Big|_0^1 + \left(-x^4 + \frac{16}{3}x^3 - 6x^2\right)\Big|_1^3$

2

$= \left(1 - \frac{16}{3} + 6\right) + (-81 + 144 - 54) - \left(-1 + \frac{16}{3} - 6\right)$

$= 7 - 5\frac{1}{3} + 9 + 7 - 5\frac{1}{3}$

$= 12\frac{1}{3}$

1

Consider the region defined by $y \le 4 - x^2$ and $y \ge -3x$.

ALL ITEMS 1 POINT EACH

SCORE: _____ / 14 PTS

$x = \pm\sqrt{4-y}$ $\quad x = -\frac{1}{3}y$

$4 - x^2 = -3x$

$0 = x^2 - 3x - 4$

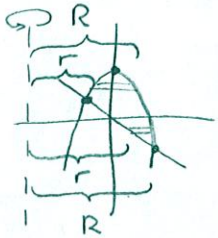
$0 = (x-4)(x+1)$

$x = -1, 4$

$y = 3 \quad y = -12$

[a] If the region is revolved around the line $x = -6$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

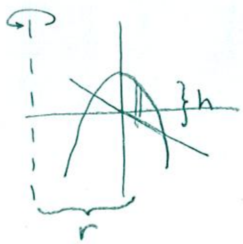
[i] using the disk or washer method **(NOTE: You do NOT need to simplify your integrand.)**



$$\pi\int_{-12}^{3}\left(\left(\sqrt{4-y}+6\right)^2-\left(-\tfrac{1}{3}y+6\right)^2\right)dy$$

$$+\ \pi\int_{3}^{4}\left(\left(\sqrt{4-y}+6\right)^2-\left(-\sqrt{4-y}+6\right)^2\right)dy$$

[ii] using the shell method **(NOTE: You do NOT need to simplify your integrand.)**



$$2\pi\int_{-1}^{4}(x+6)(4-x^2+3x)\,dx$$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$ axis are isosceles right triangles with one leg in the base. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{1}{2}\int_{-1}^{4}(4-x^2+3x)^2\,dx$$